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GEOMETRICAL ANALYSIS OF DEFORMATION SURVEYS

A. Chrzanowski

Y.Q. Chen*

J.M. Secord

Department of Surveying Engineering

University of New Brunswick

P.O. Box 4400

Fredericton, N.B., Canada

E3B 5A3

Abstract

The geometrical analysis of deformation surveys deals with the determination of the geometrical status of a deformable body — the change of its shape and dimensions. Since the deformations are usually very small and at the margin of measuring errors, very careful analysis and statistical testing of the results are required. The deformation of a body is fully described in three-dimensional space if 9 deformation parameters (6 strain components and 3 differential rotation components) can be determined at each point. These deformation parameters can be calculated from the well-known strain-displacement relationships if a displacement function representing the deformation body is known.

A methodology for finding the "best" fitting displacement function has been developed by the authors, and is known as the *UNB Generalized Method*. The Method consists of three basic processes: preliminary identification of deformation models through a trend analysis, estimation of the deformation parameters through a least-squares fitting of selected displacement functions to repeated deformation observations, and the final selection of the "best" model based on the diagnostic checking of the model and statistical testing of individual deformation parameters. The Method is applicable to any type of geometrical analysis, both in space and in time, including the detection of an unstable area and the determination of strain components and relative rigid body motions within a deformed object. It allows utilization of any type of surveying data and geotechnical measurements with configuration defects in the observation scheme. Computer program DEFNAN helps to apply the generalized method in practice. Examples of its applications are presented.

*Visiting Professor from Wuhan Technical University of Surveying and Mapping, P.R. of China (Honorary Research Associate at UNB).

1. Introduction

Expanding exploitation of mineral resources under populated areas, rapid progress in the development of large and sensitive engineering constructions, and growing interest in the study of earth crustal movements have all put new demands on the accuracy, survey methodology, and analysis of deformation measurements.

The instruments and methods of conventional geodetic and photogrammetric surveys, though still useful in collecting global deformation data, cannot satisfy all the requirements of contemporary deformation monitoring. Typical requirements are accuracies in the order of 10^{-6} and 10^{-7} , continuous monitoring with automatic recording, and telemetric data acquisition. Special instrumentation for the detection of deformations, for example, precision tiltmeters, inverted pendula, strainmeters, extensometers, mechanical and laser alignment equipment, hydrostatic levels and interferometers, is being used in structural, geotechnical, tectonic, and rock mechanics monitoring.

In order to provide a strong basis of data for any deformation analysis, the surveyor must employ new technologies and must be able to integrate all types of measurements into a comprehensive "network" of observables. This compels the surveyor to have a good understanding of the purpose and the methods of analysis of deformation surveys.

The analysis of deformations deals usually with very small deformation quantities which are at the margin of measuring errors. Therefore, a very careful accuracy analysis and statistical testing of the results are required in order to make proper decisions on the acceptance of the deformation models.

Thus the survey methods, their design and the analysis of the deformation surveys become very complex. Till now, surveyors have been little, or not at all, involved in the deformation interpretation which usually has been done by other specialists. Emphasis must be placed on the danger inherent when the surveyor, whose realm is measurement processes and the associated errors and statistical considerations, is not able to direct the interpretation of the data. Often severe misinterpretation regarding a phenomena can occur if due regard for the "quality" of the data is not given. Certainly no specialist should be the sole analyst, and it is the interaction of the survey engineer with the data user that should be encouraged. Thus a strong interaction between the survey engineer and other specialists who are in charge of the geotechnical, construction, or geophysical project is necessary during the entire life of a project.

In the past few years, more attention has been paid to the analysis of deformation surveys than ever before. In 1978, Commission 6 of the Fédération Internationale des Géomètres (FIG) created an ad hoc committee on the analysis of deformation

measurements under the chairmanship of Dr. Chrzanowski. The main task of the committee has been to compare different approaches to deformation analysis using the same measuring data with an ultimate goal to prepare a proposal for guidelines and specifications for all aspects of deformation analysis, including studies in the following items:

- (1) optimization and design of monitoring networks with geodetic and non-geodetic observables;
- (2) assessment of the observation data, detection of outliers, and systematic errors;
- (3) geometrical analysis of deformations;
- (4) physical interpretation of deformations, e.g., establishment of load-deformation relationships.

During the period 1978-1982, membership in the committee was limited to only five research centres in order to avoid difficulties and delays in the exchange of information and organization of the working meetings. The five groups, called by the names of their location (with the names of the original chief investigators in parentheses) were: Delft (J. Kok), Fredericton (A. Chrzanowski), Hannover (W. Niemeier and H. Pelzer), Karlsruhe (B. Heck and J. Van Mierlo), and Munich (W. Welsch). After the third FIG symposium on deformation surveys, which was held in Budapest in 1982, 14 more groups joined the committee. A full list of the member groups was given in Chrzanowski and Secord [1983]. At the 1986 XVIII FIG Congress in Toronto, a general theory of deformation analysis was presented, and the approaches developed by the groups of the committee were compared in the general theory [Chrzanowski and Chen, 1986].

In these notes, the authors will provide a contemporary methodology for the analysis of deformation measurements. Due to the space limitations, no detailed derivation of the formulae will be provided, but references may be consulted.

2. General Background on the Analysis of Deformation Surveys

2.1 General Classification of Deformation Analysis Methods

If acted upon by external forces (loads), any real material deforms, i.e., changes its dimensions and shape. Under the action of loads, internal stresses (force per unit area) are produced. If the stresses exceed certain critical values, the material fails (breaks). Thus the following two aspects of deformation should be distinguished in the analysis of deformation surveys:

- (1) geometrical, if we are interested only in the geometrical status of the deformable body, the change of its shape and dimensions;

- (2) physical, if we want to determine the physical status of the deformable body, the state of internal stresses, and, generally, the load-deformation relationship.

In the first case, information on the acting forces and stresses and on physical properties of the body are of no interest to the interpreter or are not available. As a final result of the geometrical analysis of deformation surveys, usually only relative displacements of discrete points are given with their variance-covariance matrix. The geometrical analysis is of particular importance when the deformable structure is supposed to satisfy certain geometrical conditions, such as verticality or the alignment of some of its components. In that case, the results of the deformation surveys are directly utilized in an adjustment of the geometrical status.

In a more refined geometrical analysis, when an overall picture of the geometrical status is required, the displacement field (or fields) for the entire body is approximated through a least-squares fitting of a selected displacement function (deformation model) into the observed displacements, as discussed in Chrzanowski et al. [1983]. The displacement field may be readily transformed into a strain field through the well-known strain-displacement relationship.

In the physical analysis of deformations, the load-deformation relationship may be modelled by using either an *empirical (statistical) method*, through a correlation of observed deformations with the observed loads; or a *deterministic method*, which utilizes information on the loads, properties of the material, and physical laws governing the stress-strain relationship.

In this presentation, only the geometrical analysis of deformation surveys will be discussed. The physical interpretation of deformations will be briefly discussed in another presentation by Chen and Chrzanowski.

2.2 Classification of Geodetic Monitoring Networks

Generally, in deformation measurements by geodetic methods, whether they are performed for monitoring engineering structures or ground subsidence in mining areas or tectonic movements, two basic types of geodetic networks are distinguished [Chrzanowski et al., 1981]:

- (1) absolute networks in which some of the points are, or are assumed to be, outside the deformable body (object) thus serving as reference points (reference network) for the determination of absolute displacements of the object points (Figure 2.1);
- (2) relative networks in which all the surveyed points are assumed to be located on the deformable body (Figure 2.2).

In the first case, the main problem of deformation analysis is to confirm the stability

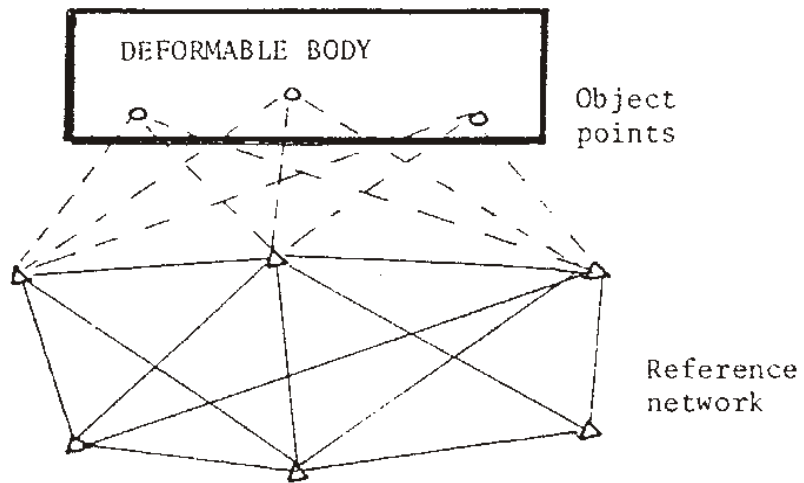


Figure 2.1 Absolute monitoring network

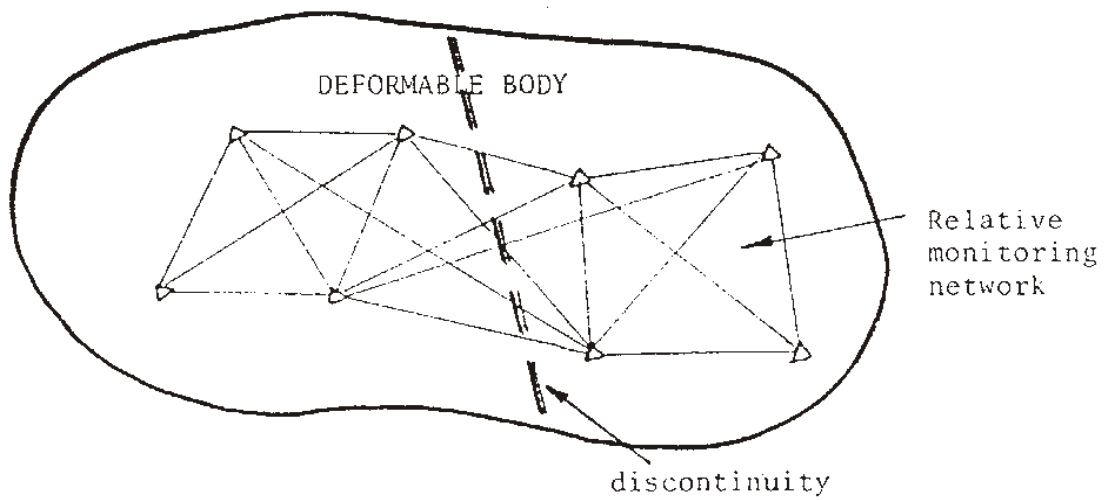


Figure 2.2 Relative monitoring network

of the reference points and to identify the possible single point displacements caused, for instance, by local surface forces and wrong monumentation of the survey markers. Once the stable reference points are identified, the determination of the geometrical state of the deformable body is rather simple.

In relative networks, deformation analysis is more complicated because, in addition to the possible single point displacements like in the reference network, all the points undergo relative movements caused by strains in the material of the body and by relative rigid translations and rotations of parts of the body if discontinuities in the material (tectonic faults, for instance) are present. The main problem in this case is to identify the deformation model. From repeated geodetic observations, it is necessary to distinguish between the deformations caused by the extension and shearing strains, by the relative rigid body displacements and by the single point displacements.

3. Deformation Modelling

3.1 Deformation Parameters

The deformation of a body is fully described in three-dimensional space if 9 deformation parameters, 6 strain components and 3 differential rotation components, can be determined at each point. In addition, components of relative rigid body motion between blocks should also be determined if discontinuities exist in the body. These deformation parameters can be calculated if a displacement function representing the deformation of the body is known. Denote the displacement function by

$$d(x, y, z; t - t_0) = \begin{bmatrix} u(x, y, z; t - t_0) \\ v(x, y, z; t - t_0) \\ w(x, y, z; t - t_0) \end{bmatrix} \quad (3.1)$$

with u, v, w as the components respectively of the displacement in the x, y, z directions, which are functions of both position and time. Then the normal strains designating elongation or compression in the directions x, y, z are calculated from:

$$\varepsilon_x = \partial u / \partial x \quad , \quad \varepsilon_y = \partial v / \partial y \quad , \quad \varepsilon_z = \partial w / \partial z \quad , \quad (3.2)$$

and the shear strains characterizing the distortion of the angles between initially corresponding lines are obtained as

$$\begin{aligned} \varepsilon_{xy} &= (\partial u / \partial y + \partial v / \partial x) / 2 \\ \varepsilon_{xz} &= (\partial u / \partial z + \partial w / \partial x) / 2 \\ \varepsilon_{yz} &= (\partial v / \partial z + \partial w / \partial y) / 2 \end{aligned} \quad (3.3)$$

The differential rotations around the x, y, z axes are expressed as

$$\begin{aligned}\omega_x &= (\partial v / \partial z - \partial w / \partial y) / 2 \\ \omega_y &= (\partial u / \partial z - \partial w / \partial x) / 2 \\ \omega_z &= (\partial u / \partial y - \partial v / \partial x) / 2 ,\end{aligned}\tag{3.4}$$

respectively. In general, the above derived quantities are time dependent and their derivatives with respect to time provide the strain rate.

Certain functions of these strain parameters, for instance, maximum strain, dilatation, pure shear, simple shear, and total shear, may also be of interest and their definitions can be found in, e.g., Sokolnikoff [1956] or Frank [1966]. Thus the main task of deformation analysis is to obtain a displacement function, which characterizes the deformation in space and in time.

3.2 Deformation Models

Since, in practice, deformation surveys are made only at discrete points, the deformation of a body must be approximated through some selected model which fits into the observation data in the best possible way. The displacement function (eqn. (3.1)) can be expressed in matrix form as:

$$\mathbf{d} = \mathbf{B} \mathbf{c} ,\tag{3.1'}$$

where \mathbf{B} is called the deformation matrix with its elements being functions of the position of the observation points and of time, and \mathbf{c} is the vector of unknown coefficients to be estimated. For illustration, examples of typical deformation models in two-dimensional space are given below.

- (1) Single point displacement or a rigid body displacement of a group of points, say, block B (Figure 3.1a) with respect to block A. The deformation model is expressed as:

$$u_A = 0 , \quad v_A = 0 \quad ; \quad u_B = a_0 \quad \text{and} \quad v_B = b_0 ,\tag{3.5}$$

where the subscripts represent all the points in the indicated blocks.

- (2) Homogeneous strain in the whole body and differential rotation (Figure 3.1b), the deformation model is linear as

$$\begin{aligned}u &= \epsilon_x x + \epsilon_{xy} y - \omega y \\ v &= \epsilon_{xy} x + \epsilon_y y + \omega x\end{aligned}\tag{3.6}$$

where the physical meaning of the coefficients is defined in eqns (3.2) to (3.4) with ω_z in eqn. (3.4) being replaced by ω .

- (3) A deformable body with one discontinuity (Figure 3.1c), say, between blocks A and B, and with different linear deformations in each block plus a rigid body displacement of B with respect to A. Then the deformation model is written as

$$\begin{aligned} u_A &= \varepsilon_{xA}x + \varepsilon_{xyA}y - \omega_A y \\ v_A &= \varepsilon_{xyA}x + \varepsilon_{yA}y + \omega_A x \end{aligned} \quad (3.7a)$$

and

$$\begin{aligned} u_B &= a_0 + \varepsilon_{xB}(x - x_0) + \varepsilon_{xyB}(y - y_0) - \omega_B(y - y_0) \\ v_B &= b_0 + \varepsilon_{xyB}(x - x_0) + \varepsilon_{yB}(y - y_0) + \omega_B(x - x_0), \end{aligned} \quad (3.7b)$$

where x_0, y_0 are the coordinates of any point in block B.

The components Δu_i and Δv_i of a total relative dislocation at any point i located on the discontinuity line between blocks A and B can be calculated as:

$$\Delta u_i = u_B(x_i, y_i) - u_A(x_i, y_i) \quad (3.8)$$

and

$$\Delta v_i = v_B(x_i, y_i) - v_A(x_i, y_i) . \quad (3.9)$$

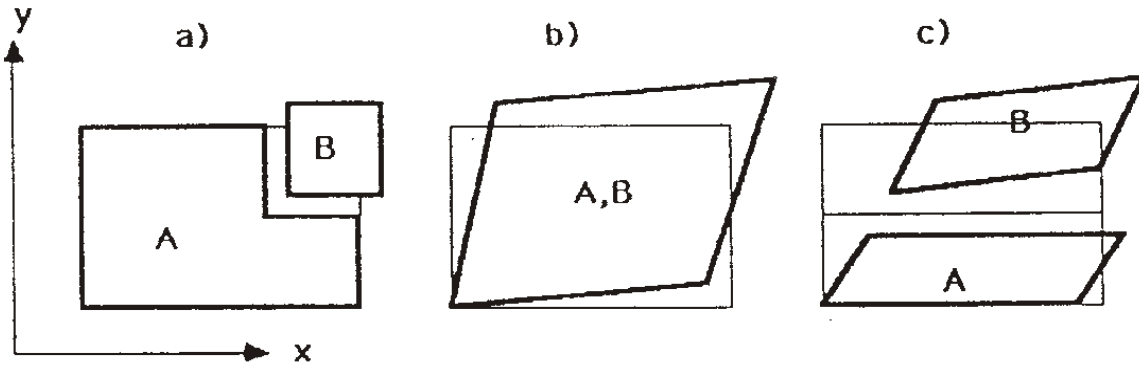


Figure. 3.1 Typical deformation models.

Usually, the actual deformation model is a combination of the above simple models or, if more complicated, it is expressed by non-linear displacement functions which require fitting of higher-order polynomials or other suitable functions.

If time dependent deformation parameters are sought, then the above deformation models will contain time variables. For instance, in the first model above (eqn. (3.5)), if the velocity (rate) and acceleration of the dislocation of block B with respect to block A are to be found, the deformation model would be

$$u_A = 0, \quad v_A = 0, \quad u_B = \dot{a}_0 t + \ddot{a}_0 t^2 \quad \text{and} \quad v_B = \dot{b}_0 t + \ddot{b}_0 t^2 \quad (3.10)$$

and, in the model of the homogeneous strain, if a linear time dependence is assumed, the model becomes:

$$u(x, y, t) = \dot{\varepsilon}_x x t + \dot{\varepsilon}_{xy} y t - \dot{\omega} y t \quad (3.11)$$

$$v(x, y, t) = \dot{\varepsilon}_{xy} x t + \dot{\varepsilon}_y y t + \dot{\omega} x t, \quad (3.12)$$

where the dot above the parameters indicates their rate (velocity) and the double dot their acceleration.

3.3 The Functional Relationship Between the Deformation Model and the Observed Quantities.

Any observation, geodetic or photogrammetric, or geotechnical measurement made in deformation surveys will contribute to the determination of deformation parameters and should be fully utilized in the analysis. The functional relationships between different observable types and the deformation model, defined in eqns (3.1) and (3.1'), are given below using a local coordinate system.

- (1) Observation of **coordinates** of point i , for instance, the coordinates derived from photogrammetric measurements or obtained using space techniques:

$$\begin{bmatrix} x_i(t) \\ y_i(t) \\ z_i(t) \end{bmatrix} = \begin{bmatrix} x_i(t_0) \\ y_i(t_0) \\ z_i(t_0) \end{bmatrix} + \begin{bmatrix} u_i \\ v_i \\ w_i \end{bmatrix}, \quad (3.13)$$

or

$$\mathbf{r}_i(t) = \mathbf{r}_i(t_0) + \mathbf{d}_i = \mathbf{r}_i(t_0) + \mathbf{Bc}, \quad (3.13')$$

where \mathbf{r}_i is the position vector of point i , and the others are defined in eqn. (2.1).

- (2) Observation of **coordinate differences** between points i and j , e.g., height difference (levelling) observation, pendulum (displacement) measurement, and alignment survey:

$$\begin{bmatrix} x_j(t) - x_i(t) \\ y_j(t) - y_i(t) \\ z_j(t) - z_i(t) \end{bmatrix} = \begin{bmatrix} x_j(t_0) - x_i(t_0) \\ y_j(t_0) - y_i(t_0) \\ z_j(t_0) - z_i(t_0) \end{bmatrix} + \begin{bmatrix} u_j - u_i \\ v_j - v_i \\ w_j - w_i \end{bmatrix} \quad (3.14)$$

or

$$\mathbf{r}_j(t) - \mathbf{r}_i(t) = \mathbf{r}_j(t_0) - \mathbf{r}_i(t_0) + \{B(x_j, y_j, z_j; t-t_0) - B(x_i, y_i, z_i; t-t_0)\} \mathbf{c}. \quad (3.14')$$

If the components of the displacement obtained from a pendulum observation do not coincide with the coordinate axes, a transformation to the common coordinate system has to be performed. Similarly, a coordinate transformation may be required in alignment surveys which provide a transverse displacement of a point with respect to a straight line defined by two base points.

- (3) Observation of **azimuth** from point i to point j

$$\alpha_{ij}(t) = \alpha_{ij}(t_0) + [(-\cos\alpha_{ij}) / (S_{ij}\cos\beta_{ij}), (\sin\alpha_{ij}) / (S_{ij}\cos\beta_{ij})] \begin{bmatrix} u_j - u_i \\ v_j - v_i \end{bmatrix} \quad (3.15)$$

where β_{ij} and S_{ij} are the vertical angle and spatial distance from point i to point j, respectively. The observation of a horizontal angle is expressed as the difference of two azimuths.

- (4) Observation of the **distance** between points i and j:

$$S_{ij}(t) = S_{ij}(t_0) + (\cos\beta_{ij} \sin\alpha_{ij}, \cos\beta_{ij} \cos\alpha_{ij}, \sin\beta_{ij}) \begin{bmatrix} u_j - u_i \\ v_j - v_i \\ w_j - w_i \end{bmatrix} \quad (3.16)$$

- (5) Observation of **strain** along the azimuth α and vertical angle β at point i:

$$\epsilon(t) = \epsilon(t_0) + \mathbf{p}^T \mathbf{E} \mathbf{p}, \quad (3.17)$$

where

$$\mathbf{p}^T = (\cos\beta \sin\alpha, \cos\beta \cos\alpha, \sin\beta)$$

$$\mathbf{E} = \begin{bmatrix} \partial u/\partial x & \partial u/\partial y & \partial u/\partial z \\ \partial v/\partial x & \partial v/\partial y & \partial v/\partial z \\ \partial w/\partial x & \partial w/\partial y & \partial w/\partial z \end{bmatrix}$$

- (6) Observation of a **vertical angle** at point i to point j:

$$\beta_{ij}(t) = \beta_{ij}(t_0) + \begin{pmatrix} \frac{\sin\beta_{ij} \sin\alpha_{ij}}{S_{ij}}, -\frac{\sin\beta_{ij} \cos\alpha_{ij}}{S_{ij}}, \frac{\cos\beta_{ij}}{S_{ij}} \end{pmatrix} \begin{bmatrix} u_j - u_i \\ v_j - v_i \\ w_j - w_i \end{bmatrix} \quad (3.18)$$

- (7) Observation of a **horizontal tiltmeter**:

$$\tau(t) = \tau(t_0) + (\partial w/\partial x) \sin\alpha + (\partial w/\partial y) \cos\alpha \quad (3.19)$$

where α is the orientation of the tiltmeter.

In the above formulac, the quantities u, v, w and their derivatives are replaced by the deformation model which is explicitly expressed in eqn. (3.1'). Thus all the observations are functions of the unknown coefficients c .

4. Remarks on the Adjustment of Monitoring Networks

As discussed in section 3.3, the deformation parameters can be directly estimated from the observations. However, if the observation scheme includes a complete geodetic network (without configuration defect), it is recommended that the whole procedure of deformation analysis be separated into two parts:

- (1) adjustment of the network for each campaign;
- (2) fitting of a deformation model into displacements (quasi-observables) calculated from paired differences in the adjusted coordinates.

The adjustment process provides an opportunity for detecting outliers and systematic errors in the observations, as well as for the evaluation of the quality of the observations. Appendices I and II give a brief review on the detection of outliers and on the assessment of the observations, respectively, using methods developed at UNB [Chen 1983; Chen and Chrzanowski, 1985; Chen et al., 1986].

If subjected to the proper transformation (see section 5), the displacements calculated from the adjusted coordinates give a picture of the deformation pattern and help in the identification (trend analysis) of the deformation model.

For the sake of completeness in the discussion of deformation analysis, some remarks on the adjustment of monitoring networks are given below.

Deformation monitoring networks are mostly free networks, suffering from datum defects. Consider the n -vector of observations l with dispersion measured by $\sigma_0^2 Q$ in a monitoring network such that

$$l + v = A_y y + A_x x, \quad (4.1)$$

where v is the n -vector of residuals; x is the vector of coordinates of surveyed points; y is the vector of nuisance parameters, e.g., the orientation unknown for each round of directions; and A_y, A_x are corresponding configuration matrices. The least-squares criterion leads to the normal equations:

$$\begin{bmatrix} A_y^T Q^{-1} A_y & A_y^T Q^{-1} A_x \\ A_x^T Q^{-1} A_y & A_x^T Q^{-1} A_x \end{bmatrix} \begin{bmatrix} y \\ x \end{bmatrix} = \begin{bmatrix} A_y^T Q^{-1} l \\ A_x^T Q^{-1} l \end{bmatrix}. \quad (4.2)$$

Eliminating vector y , one gets

$$\begin{aligned} A_x^T [Q^{-1} - Q^{-1} A_y (A_y^T Q^{-1} A_y)^{-1} A_y^T Q^{-1}] A_x x = \\ = A_y^T [Q^{-1} - Q^{-1} A_y (A_y^T Q^{-1} A_y)^{-1} A_y^T Q^{-1}] l, \end{aligned} \quad (4.3)$$

or, more compactly, as

$$N x = w \quad (4.3')$$

Due to datum defects in the monitoring network, the coefficient matrix, N , of the normal equations is singular. Therefore, one must define datum equations to solve for x . Let $D^T x = 0$ be the datum equations in which the rank of matrix D is equal to the number of datum defects in the network. Then, the solution of eqn (4.3') becomes

$$\hat{x} = N_D^- w \quad , \quad Q_x = N_D^- \quad (4.4)$$

with

$$N_D^- = (N + DD^T)^{-1} - H(H^T D D^T H)^{-1} H^T \quad (4.5)$$

The matrix H generates the null space of matrix N , i.e., $NH=0$. For example, for a triangulation network, matrix H reads as:

$$H^T = \begin{bmatrix} 1 & 0 & 1 & 0 & \dots & 1 & 0 \\ 0 & 1 & 0 & 1 & \dots & 0 & 1 \\ -y_1^0 & x_1^0 & -y_2^0 & x_2^0 & \dots & -y_m^0 & x_m^0 \\ x_1^0 & y_1^0 & x_2^0 & y_2^0 & \dots & x_m^0 & y_m^0 \end{bmatrix} \quad (4.6)$$

where x_i^0, y_i^0 are the coordinate components of point i with respect to the centroid of the network. For a trilateration network, the last row of H^T in eqn. (4.6) disappears.

In the general case of a three-dimensional network consisting of m points, one expression of matrix H has, for the maximal case of seven datum defects, the structure:

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 & z_1^0 & -y_1^0 & x_1^0 \\ 0 & 1 & 0 & -z_1^0 & 0 & x_1^0 & y_1^0 \\ 0 & 0 & 1 & y_1^0 & -x_1^0 & 0 & z_1^0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 1 & 0 & 0 & 0 & z_m^0 & -y_m^0 & x_m^0 \\ 0 & 1 & 0 & -z_m^0 & 0 & x_m^0 & y_m^0 \\ 0 & 0 & 1 & y_m^0 & -x_m^0 & 0 & z_m^0 \end{bmatrix} (3m) \times (7) \quad (4.7)$$

where x_i^0, y_i^0 , and z_i^0 are the approximate coordinates of the points in the directions x, y , and z , respectively, with respect to the centroid of the network. The first three columns of matrix H correspond to the translation of the network in the directions x, y, z ; the second three columns take care of the rotation of the network at the centroid about the x, y, z axes, respectively; the last column accounts for the change in scale.

The solution of eqn. (4.3') with respect to the datum $\mathbf{D}^T \mathbf{x} = \mathbf{0}$ can also be realized through a similarity transformation from any solutions (say \mathbf{x}_u) as

$$\mathbf{x} = \mathbf{S} \mathbf{x}_u, \quad \mathbf{Q}_x = \mathbf{S} \mathbf{Q}_{x_u} \mathbf{S}^T \quad (4.8)$$

with

$$\mathbf{S} = (\mathbf{I} - \mathbf{H}(\mathbf{D}^T \mathbf{H})^{-1} \mathbf{D}^T) = \mathbf{I} - \mathbf{H}(\mathbf{H}^T \mathbf{W} \mathbf{H})^{-1} \mathbf{H}^T \mathbf{W}, \quad (4.9)$$

where $\mathbf{W} = \mathbf{D}(\mathbf{D}^T \mathbf{D})^{-1} \mathbf{D}^T$. The matrix \mathbf{W} in eqn. (4.9) can be interpreted as a weight matrix in the definition of the datum. If all the points in the network are of the same importance in defining the datum, then $\mathbf{W} = \mathbf{I}$ and eqn. (4.8) becomes the inner constraints solution.

If only some points are used to define the datum, then the other points are given zero weight. For more details, refer to Chen [1983].

From the adjustment, the a posteriori variance factor $\hat{\sigma}_0^2$ is calculated from

$$\hat{\sigma}_0^2 = \mathbf{v}^T \mathbf{Q}^{-1} \hat{\mathbf{v}} / df, \quad (4.10)$$

with the degrees of freedom $df = (n - u_x - u_y + d)$, where \mathbf{v} is the vector of estimated residuals, u_x, u_y are the numbers of unknown parameters x and y , respectively, and d is the number of datum defects. When there are k a posteriori variance factors σ_i^2 ($i=1, \dots, k$) with the degrees of freedom df_i from the adjustment of k epochs of observations, the pooled a posteriori variance factor can be calculated from

$$\hat{\sigma}_0^2 = \left(\sum_i^k df_i \hat{\sigma}_i^2 \right) / \left(\sum df_i \right), \quad (4.11)$$

if the Bartlett test [Bjerhammar, 1973] allows the non-rejection of the null hypothesis $H_0 : \sigma_1^2 = \sigma_2^2 = \dots = \sigma_k^2$.

5. Identification of Deformation Models

Generally, the analysis of deformation surveys consists of three basic processes:

- (1) preliminary identification of the deformation model;
- (2) estimation of the deformation parameters;
- (3) diagnostic checking of the deformation models and the final selection of the "best" model.

Identification procedures are applied to a set of data to indicate the kind of deformation that warrants further investigation. After a tentative formulation of the deformation trend, estimates of deformation parameters or the coefficients of the models are obtained using the least-squares technique. After the parameters have been estimated, diagnostic checks are performed to determine the adequacy of the fitted model or to indicate

potential improvements. Those three processes necessarily overlap and should be performed as an iterative three-step procedure.

When the deformation observations are scattered in time, a simultaneous handling of all the observations in the deformation analysis may be necessary (see section 6). The identification of the deformation model in such cases may be difficult unless the model can be assumed from a priori knowledge of the deformation mechanism. In practice, the observations are usually grouped in distinct epochs of time. Then performing the analysis on pairs of epochs is preferred to the direct simultaneous analysis of all the epochs. The analysis of pairs of epochs has the following advantages:

- (1) Single point movement in a reference network does not usually follow certain time functions and, therefore, the main interest lies in the localization of unstable points between two epochs of time.
- (2) An analyst of deformation measurements is often curious about what happened to the deformable body between the most recent surveying campaign and the previous one.
- (3) Through the analysis of successive pairs of epochs of observations, the deformation trend in the time domain will be recognized.

An important step in the analysis of pairs of epochs of observations is to identify the deformation pattern in the space domain. Moreover, if the deformation is postulated to be of a linear nature in time, then all the observations made at different epochs of time can be reduced to the observed rate of change of the observation.

Here, a method developed at the U.S. Geological Survey [Prescott et al., 1981] should be mentioned. In their method, all the observations of each line in a trilateration network are plotted against time, and then a linear time function (without precluding the possibility of nonlinearity) is fitted to each of the plots. The slope of each fitted straight line is an estimate of the average rate at which the line was changing during the time period covered by the observations. The standard deviation in the rate is also calculated. Then the problem reduces to the estimation of the deformation rate. Therefore, analysis of multi-epoch observations becomes the estimation of the deformation rate. In this case, the main task is again to identify the deformation pattern in space.

As already mentioned, the selection of deformation models may be based on a priori information or on trend analysis from the displacement pattern. If a monitoring network suffers from datum defects, which is usually the case, a method of iterative weighted transformation [Chen, 1983] can be used to yield the "best" picture of the displacement field, as discussed below.

When comparing two campaigns, the vector of displacements and its cofactor matrix

are calculated as

$$\mathbf{d} = \hat{\mathbf{x}}_2 - \hat{\mathbf{x}}_1, \quad \mathbf{Q}_d = \mathbf{Q}_{\hat{\mathbf{x}}_1} + \mathbf{Q}_{\hat{\mathbf{x}}_2} \quad \text{with } \hat{\sigma}_0^2 \text{ from eqn. (4.11).} \quad (5.1)$$

Because unstable points are not identified, the displacements calculated from eqn. (5.1) may be biased by a pre-selected datum or by a different datum definition in the adjustment of two campaign observations. A typical example for the latter case is the monitoring scheme in which a triangulation network was used in the first campaign and a trilateration or triangulation network in the second campaign.

To overcome this problem, a method of iterative weighted transformation has been developed. Let \mathbf{d}_1 and \mathbf{Q}_{d1} be calculated from eqn. (5.1). The transformation of \mathbf{d}_1 into another datum is computed from eqns. (4.8) and (4.9) as

$$\mathbf{d}_{k+1} = (\mathbf{I} - \mathbf{H}(\mathbf{H}^T \mathbf{W} \mathbf{H})^{-1} \mathbf{H}^T \mathbf{W}) \mathbf{d}_k = \mathbf{S}_k \mathbf{d}_k. \quad (5.2)$$

At the outset, the weight matrix \mathbf{W} is taken as the identity, then in the $(k+1)$ th transformation, the weight matrix is defined as

$$\mathbf{W} = \text{diag}\{1/|d_i(k)|\}, \quad (5.3)$$

where $d_i(k)$ is the i^{th} component of the vector \mathbf{d}_k after the k^{th} iteration. The iterative procedure continues until the differences between the successive transformed displacements (i.e., $\mathbf{d}_{k+1} - \mathbf{d}_k$) approach zero. During this procedure, some $d_i(k)$ may approach zero causing numerical instabilities because $W_i = 1/|d_i(k)|$ becomes very large. Thus, a lower bound is set. When $|d_i(k)|$ is smaller than the lower bound, its weight is set to zero. If in the following iterations the $d_i(k+1)$ becomes significantly large again, the weights can be changed accordingly. The method provides a datum which is robust to unstable reference points giving an unbiased depiction of displacements. In the last iteration, say $(k+1)^{\text{th}}$, the cofactor matrix should also be calculated as:

$$\mathbf{Q}_{d_{k+1}} = \mathbf{S}_k \mathbf{Q}_{d1} \mathbf{S}_k^T. \quad (5.4)$$

Comparing the displacements of each point against its confidence ellipse, one can identify the reference points which are most probably unstable. The following are two examples used to illustrate the method.

The first example is a simulated relative geodetic network across a fault line (Figure. 5.1), where only directions were measured in the first campaign and both directions and distances were measured in the second. A 200 mm relative movement in the y direction of block B with respect to Block A was introduced. Without considering measuring errors, the displacement fields using the proposed method and the method of the inner constraints solution are portrayed in Figures. 5.2 and 5.3, respectively, coupled with the real displacement pattern in dashed lines. As one can see, the displacement field obtained from the method of iterative weighted transformation is much closer to the real situation, compared with the constraint solution.

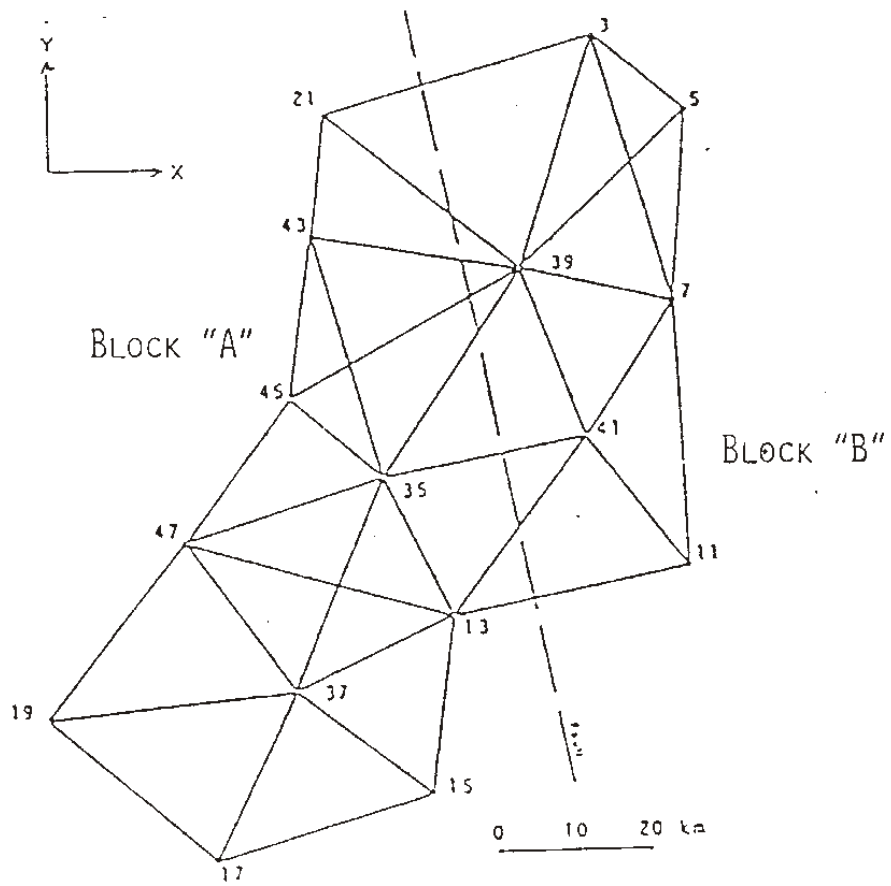


Figure 5.1 Typical campaign of the simulated relative geodetic network.

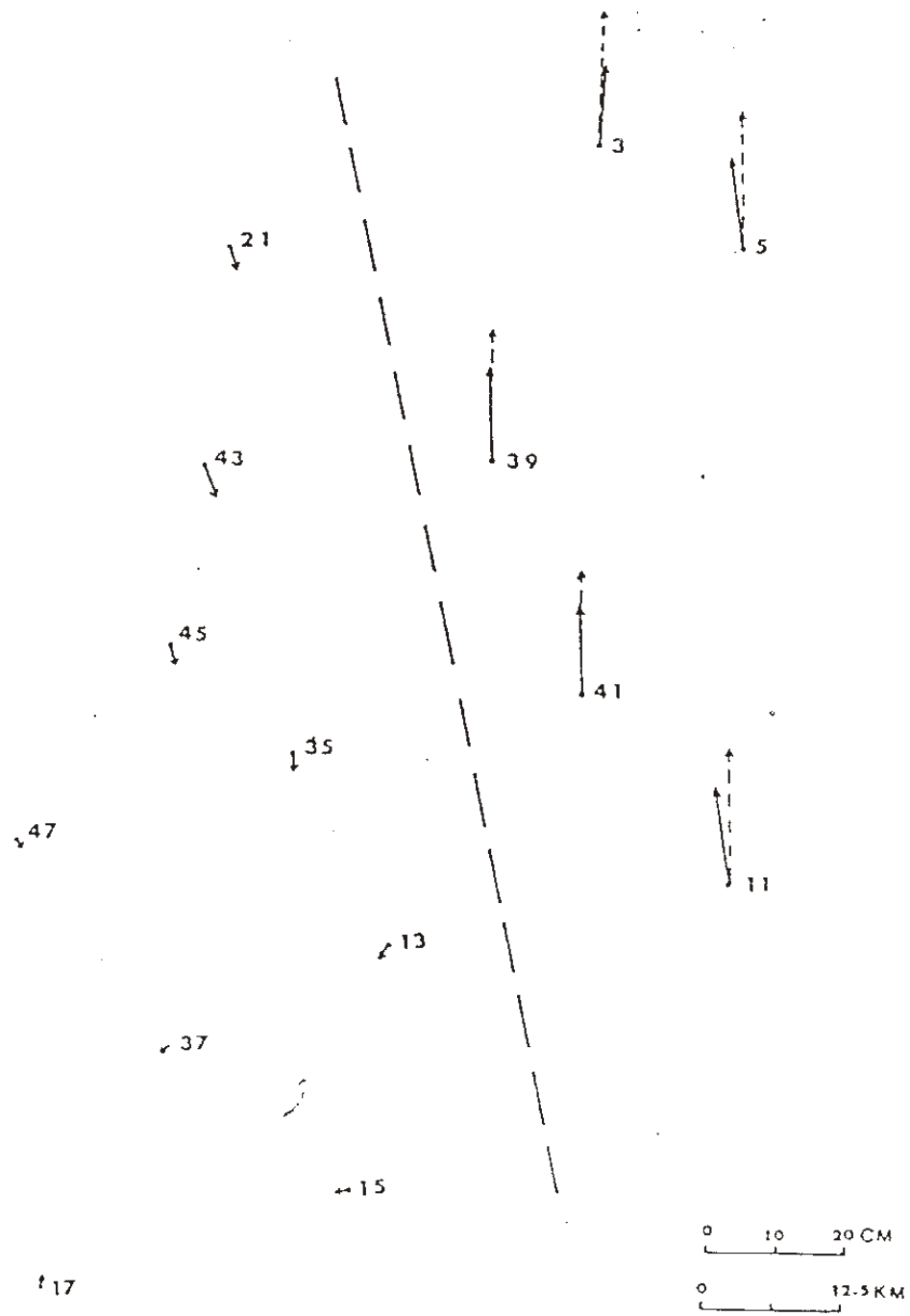


Figure 5.2 Simulated relative geodetic network—Displacement field after the iterative weighted transformation (solid) versus the actual displacements (broken).

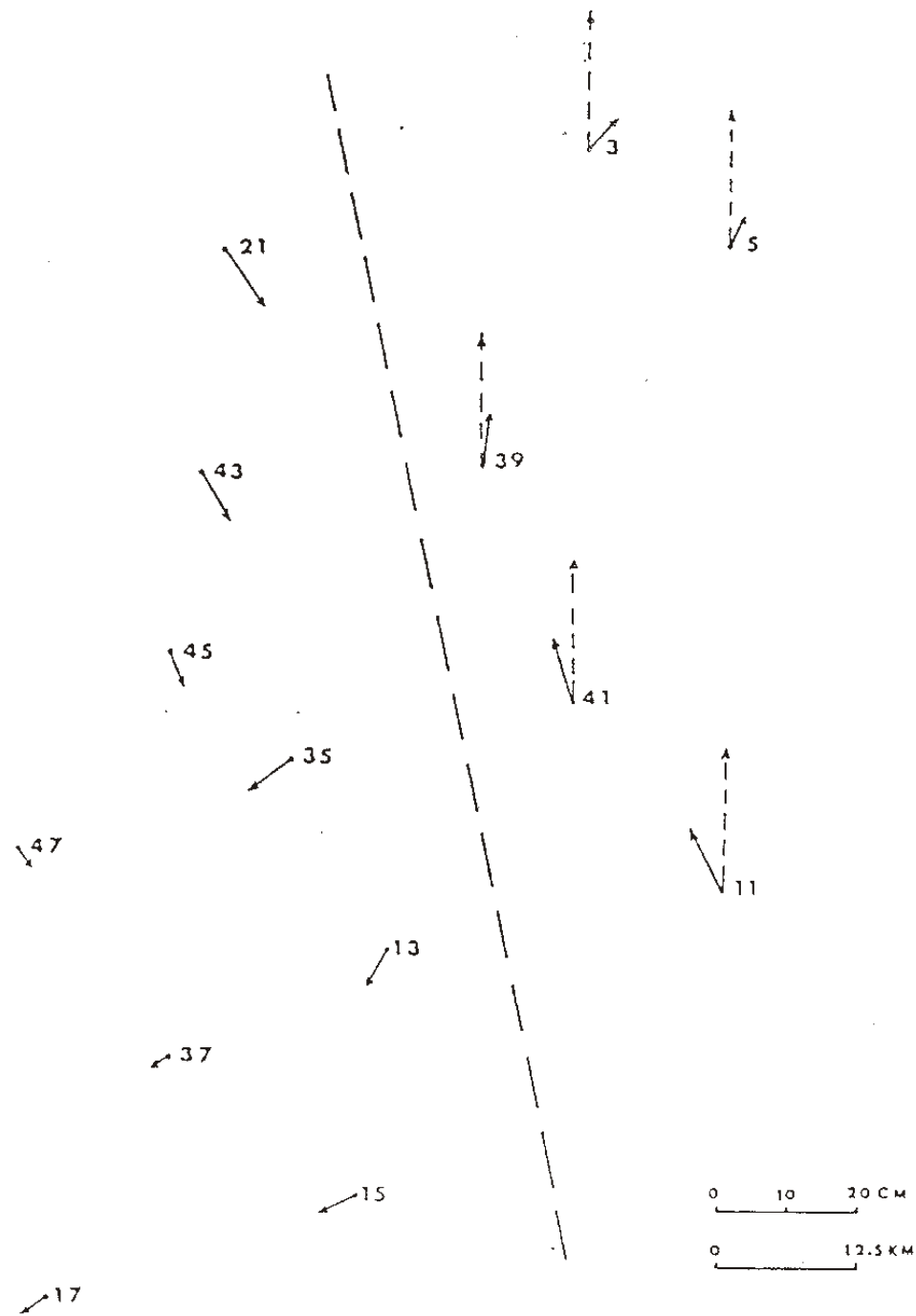


Figure 5.3 Simulated relative geodetic network—Displacement field under inner constraints (solid) versus the actual displacements (broken)

In the second example, the method is applied to a dam monitoring network consisting of a reference network of six stations from which a number of targeted points on the dam were positioned. The same network in actual conditions is shown in the example of section 10 (Figure 10.1) A simulated movement in steps of 1 mm was introduced to station 5 in the y direction between the successive survey campaigns (total of 11 campaigns). The displacement field coupled with the error ellipses at 95% confidence level were calculated using the method of iterative weighted transformation and the method of the inner constraints solution. Table 1 summarizes the results of the identification of the points suspected as being unstable because their displacements extended beyond the confidence region at 95%.

TABLE 1.
Points outside the 95% confidence region after introducing simulated displacements to station 5.

Suspected Unstable point*	Accumulated simulated displacement of station 5 in mm									
	1	2	3	4	5	6	7	8	9	10
1							0	0	0	0
2										0
3					0	0	0	0	0	0
4				0	0	0	0	0	0	0
5	X0	X0	X0	X0	X0	X0	X0	X0	X0	X0
6									0	0

* X: using the method of iterative weighted transformation.

0: using the method of the inner constraints solution.

It is clear from Table 1 that the method of iterative weighted transformation identified the unstable point correctly, while the method of inner constraints solution declared more suspected unstable points.

The method of the weighted transformation is flexible. If some points are more likely to move, a weight of zero is assigned to each of these points during the iterative process. For example, if the points on one side of a tectonic fault may likely move with

respect to the points on the other side, then only the points on the one side are used to define a datum.

6. Estimation of Deformation Models

Let y_i ($i=1,2,\dots, k$) be the vector of observations in epoch i , including quasi-observations (e.g., the coordinates of points from an adjustment of geodetic network of photogrammetric surveys), geotechnical measurements (using strainmeters, extensometers, tiltmeters, etc.), and individual geodetic observations, and P_i be the weight matrix of y_i . The weight matrix for the coordinates of points estimated from eqn. (3.3') is taken as N , and for the other observed quantities it is taken in the conventional way as the inverse of the cofactor matrix. Because of datum defects and possible configuration defects in a monitoring network, the weight matrix P_i is, in general, considered as being singular [Chrzanowski et al., 1983]. Determination of the coefficients of a deformation model, $d(x, y, z; t-t_1) = B(x, y, z; t-t_1)c$, is based on the following functional relations:

$$\begin{array}{c}
 \left[\begin{array}{c} y_1 \\ y_2 \\ \vdots \\ y_k \end{array} \right] + \left[\begin{array}{c} \delta_1 \\ \delta_2 \\ \vdots \\ \delta_k \end{array} \right] = \left[\begin{array}{c} I \\ I \\ \vdots \\ I \end{array} \right] \cdot \left[\begin{array}{c} 0 \\ \tilde{B}_2 \\ \vdots \\ \tilde{B}_k \end{array} \right] + \left[\begin{array}{c} \xi \\ \vdots \\ c \end{array} \right] \quad (6.1)
 \end{array}$$

with weight matrix $P = \text{diag}\{P_1, P_2, \dots, P_k\}$, ξ is the expected value of y_1 , and δ_i is a vector of residuals after fitting the deformation model to the y_i ; matrix \tilde{B}_i is a function of the position of points and time. If y_i is the vector of coordinates, then $\tilde{B}_i = B_i$. If y_i is the vector of observations rather than of the coordinates of points, $\tilde{B}_i = AB_i$, where matrix A is the transformation matrix (or configuration matrix) relating the observations to the coordinates. In order to keep the same population of vector y_i in each epoch, dummy observations with zero weight are put in the place of observations missing in campaign i in the vector y_i . Applying the principle of least squares to model (6.1), the normal equations read:

$$\begin{array}{ccc}
\begin{array}{c} k \\ \left[\sum P_i \right. \\ | 1 \\ | \\ | k \\ \left. \sum \tilde{B}_i^T P_i \right] \\ 2 \end{array} &
\begin{array}{c} k \\ \sum P_i \tilde{B}_i \\ | | | \\ | | | \\ \sum \tilde{B}_i^T P_i \tilde{B}_i \\ | | | \\ 2 \end{array} &
\begin{array}{c} k \\ \left[\xi \right] \\ | | | \\ | | | \\ \left[c \right] \\ 2 \end{array} = \begin{array}{c} k \\ \left[\sum P_i y_i \right. \\ | 1 \\ | \\ | k \\ \left. \sum \tilde{B}_i^T P_i y_i \right] \\ 2 \end{array}
\end{array} \quad (6.2)$$

The coefficient matrix of the normal eqns. (6.2) may be singular with rank defects

$$\text{rd} \left\{ \sum_1^k P_i \right\} = d \quad (6.3)$$

which is equal to the number of remaining datum and configuration defects not determined in at least one epoch.

Eliminating ξ from eqn. (6.2) allows the vector c and its accuracy to be calculated from:

$$\hat{c} = N_c^{-1} \left[\sum_2^k \tilde{B}_i^T P_i y_i - \sum_2^k \tilde{B}_i^T P_i \left(\sum_1^k P_i \right)^{-1} \sum_1^k P_i y_i \right] \quad (6.4)$$

and

$$\hat{C}_c = \sigma_0^2 N_c^{-1} = \sigma_0^2 \left[\sum_2^k \tilde{B}_i^T P_i \tilde{B}_i - \sum_2^k \tilde{B}_i^T P_i \left(\sum_1^k P_i \right)^{-1} \sum_2^k P_i \tilde{B}_i \right]^{-1} \quad (6.5)$$

To serve as the a priori variance factor σ_0^2 , the pooled variance (a posteriori to the campaign adjustments) factor obtained from eqn. (4.11) is used.

As was already mentioned, an indispensable step in the deformation analysis is the analysis of pairs of epochs of observations. In this special case, for each pair of epochs, eqn. (6.4) reduces to:

$$\hat{c} = (\tilde{B}^T P_{\Delta y} \tilde{B})^{-1} \tilde{B}^T P_{\Delta y} \Delta y, \quad (6.6)$$

in which

$$\Delta y = y_2 - y_1$$

$$P_{\Delta y} = P_2 - P_2(P_1 + P_2)^{-1} P_2$$

If y_i stands for observations l_i , solution (6.6) becomes:

$$\hat{c} = [B^T A^T P_{\Delta t} A B]^{-1} B^T A^T P_{\Delta t} (l_2 - l_1) \quad (6.7)$$

with

$$\mathbf{P}_{\Delta l} = (\mathbf{Q}_1 + \mathbf{Q}_2)^{-1} .$$

This is called the "observation approach." If y_i stands for the estimated coordinates x_i from eqn. (4.3'), then

$$\hat{\mathbf{c}} = (\mathbf{B}^T \mathbf{P}_d \mathbf{B})^{-1} \mathbf{B}^T \mathbf{P}_d \mathbf{d} , \quad (6.8)$$

with $\mathbf{d} = (x_2 - x_1)$ and $\mathbf{P}_d = \mathbf{N}_1(\mathbf{N}_1 + \mathbf{N}_2)^{-1} \mathbf{N}_2$. Then it is named the "displacement approach."

7. Assessment of the Deformation Models

7.1 General Remarks on Statistical Testing

Analysis of deformation surveys involves several tests of hypotheses. Consider the observation equations:

$$\mathbf{l} = \mathbf{A}\mathbf{x} + \mathbf{v} , \quad (7.1)$$

where \mathbf{l} is a vector of n observations with normal distribution and dispersion $\sigma_0^2 \mathbf{Q}$, \mathbf{x} is a vector of u unknown parameters, \mathbf{A} is the design matrix (or configuration matrix), \mathbf{v} is the vector of residuals, σ_0^2 is the a priori variance factor, and \mathbf{Q} is a cofactor matrix so that the observations have a weight matrix $\mathbf{P} = \mathbf{Q}^{-1}$. If the null hypothesis $H_0 : \mathbf{H}\mathbf{x} = \mathbf{w}$ is to be tested against an alternative $H_a : \mathbf{H}\mathbf{x} \neq \mathbf{w}$, then the test statistic can be obtained by imposing the constraints $\mathbf{H}\mathbf{x} = \mathbf{w}$ on the parameters \mathbf{x} . Under the null hypothesis, model (7.1) becomes:

$$\left\{ \begin{array}{l} \mathbf{l} = \mathbf{A}\mathbf{x} + \mathbf{v} \quad \text{with } \sigma_0^2 \mathbf{Q} \\ \mathbf{H}\mathbf{x} = \mathbf{w} \end{array} \right. \quad (7.2)$$

The acceptance of the null hypothesis at a certain significant level α is ensured by the inequality:

$$T = [((R_1 - R_0) / R_0)(df/(df_1 - df))] \leq F(\alpha; df_1 - df, df) , \quad (7.3)$$

or, equivalently,

$$T = [(R_1 - R_0)/R_1](df_1/(df_1 - df)) \leq (df_1 F(\alpha; df_1 - df, df))/(df + (df_1 - df) F(\alpha; df_1 - df, df)) , \quad (7.4)$$

where R_0 and R_1 are the quadratic forms of the residuals from the adjustment of model (7.1) and model (7.2), respectively. The corresponding degrees of freedom are:

$$df = n - \text{rank}\{\mathbf{A}\} ,$$

and

$$df_1 = n - \text{rank}\{A^T : H^T\} + \text{rank}\{H^T\} .$$

In the practice of hypothesis testing, $(R_1 - R_0)$ and R_0 in the statistics (7.3) and (7.4) can be calculated, depending on the problem at hand, in three different ways [Chen, 1983]:

- (i) from separate adjustments of model (7.1) and model (7.2),
- (ii) from the adjustment of model (7.1),

$$R_1 - R_0 = (Hx - w)^T (H(A^T Q^{-1} A)^{-1} H^T)^{-1} (Hx - w) \quad (7.5)$$

- (iii) from the adjustment of model (7.2)

$$R_1 - R_0 = v^T Q^{-1} A_2 (A_2^T Q^{-1} Q_v Q^{-1} A_2)^{-1} A_2^T Q^{-1} v , \quad (7.6)$$

where v and Q_v are the vector of residuals and its cofactor matrix, respectively, and matrix A_2 generates the space whose union with the solution space of model (7.2) equals the solution space of model (7.1).

7.2 Assessment and Final Selection of the Deformation Model

The global appropriateness of a deformation model can be tested using a quadratic function of the residuals δ_i in eqn. (6.1) as

$$\Delta R = \sum_1^k \delta_i^T P_i \delta_i , \quad (7.7)$$

where the notation has been defined in section 6. The quantity ΔR follows a chi-squared distribution with degrees of freedom being

$$df_c = \sum_1^k r\{P_i\} - u + d \quad (7.8)$$

where u is the dimension of the vector of unknowns $(\xi^T : c^T)$ of eqn. (6.1), $r\{P_i\}$ is the rank of matrix P_i , and d has been defined in eqn. (6.3). If the following inequalities hold:

$$\Delta R \leq \sigma_0^2 \chi^2(df_c; \alpha) , \quad (7.9)$$

when the a priori variance factor is known, or

$$\Delta R \leq \hat{\sigma}_0^2 df_c F(df_c, df; \alpha) , \quad (7.10)$$

when the pooled variance factor is used and $df = f_1 + \dots + f_k$, as defined in eqn. (4.11), then the deformation model is globally acceptable at the $(1-\alpha)$ confidence level. When $c=0$, the test statistic (7.9) or (7.10) can be regarded as an extension of the global congruency test, which originated from Pelzer [1971].

The significance of the individual parameter \hat{c}_i or a group of u_i parameters, \hat{c}_i which is a subset of \hat{c} , is revealed by testing the null hypothesis $H_0 : c_i=0$ or $c_i=0$ versus the alternative hypothesis $H_a : c_i \neq 0$ or $c_i \neq 0$. Their significances are indicated by

$$\hat{c}_i^2 / (\sigma_0^2 q_{ii}) \geq F(1, df; \alpha) \quad (7.11)$$

and

$$\hat{\mathbf{c}}_i^T \mathbf{Q}_{ci}^{-1} \hat{\mathbf{c}}_i / (\sigma_o^2 u_i) \geq F(u_i, df; \alpha) \quad (7.12)$$

where q_{ii} is the i^{th} diagonal element of $\mathbf{Q}_{\hat{\mathbf{c}}}$ and \mathbf{Q}_{ci} is a submatrix of $\mathbf{Q}_{\hat{\mathbf{c}}}$. If the global test fails, localization in time domain or in space domain should be performed. Displaying the residuals will help in improving the model.

Since more than one of several possible models could fit the data reasonably well, the authors have set the following criteria for selection of the "best model":

- (1) the model passes the global statistical test and all parameters are significant beyond some level of α as 0.10 or 0.05,
- (2) if more than one model satisfies the above criteria, then the model with the fewest parameters is selected.
- (3) if no model satisfies the criteria of (1), then physically-based rationale and minimum error of fit are used.

8. Summary of the Generalized Method of Geometrical Analysis

The presented approach to the geometrical analysis of deformation surveys has been named by the authors the *UNB Generalized Method*.

As shown in the previous sections, the method is applicable to any type of geometrical analysis, both in space and in time, including the detection of an unstable area and the determination of strain components and relative rigid body motion within a deformed object. It allows utilization of different types of surveying data and geotechnical measurements. In practical application, the approach consists of three basic processes: identification of deformation models; estimation of the deformation parameters; diagnostic checking of the models and the final selection of the "best" model.

The analysis procedures using the approach can be summarized in the following steps:

- (1) Assessment of the observations using the minimum norm quadratic unbiased estimation (MINQE) principle (Appendix I) to obtain the variances of observations and possible correlations of the observations within one epoch or between epochs, if the a priori values are not available.
- (2) Separate adjustment of each epoch of geodetic or photogrammetric observations, if such are available, for detection of outliers (Appendix II) and systematic errors. If correlations of the observations between epochs are not negligible, then simultaneous adjustment of multiple epochs of observations is required.

Step 1 and 2 overlap because the existence of outliers and systematic errors will influence

the estimated variances and covariances and adopted variances and covariances of the observations will affect outlier detection.

- (3) Comparison of pairs of epochs; selection of deformation models based on a priori considerations and trend analysis from the displacement pattern if such is available from the observations. If a monitoring network suffers from datum defects, the method of iterative weighted transformation is used to yield the "best" picture of the displacement pattern.
- (4) Estimation of the coefficients of deformation models and their covariance using all available information.
- (5) Global test on the deformation model; testing groups of the coefficients or an individual one for significance.

The above three steps should be considered as an iterative three-step procedure, so they necessarily overlap.

- (6) Simultaneous estimation of the coefficients of the deformation model in space and in time if the analysis of pairs of epochs of observations suggests that it is worth doing.

This simultaneous estimation must be performed if the observations are scattered in time. The iterative three-step procedure is still valid. The possible deformation models can be selected either based on a priori considerations or by plotting the observations versus time for trend analysis.

- (7) Comparison of the models and choice of the "best" model. Since more than one of several possible models could fit the data reasonably well, the "best" model is selected according to the criteria:
 - (a) the model passes the global statistical test at an acceptable probability;
 - (b) if more than one model passes the global test, then the model with the fewest significant coefficients is selected.
 - (c) if the two above criteria cannot be satisfied, then rationale based on physical ground and minimal error of fit is used.
- (8) Calculation of the desired deformation characteristics and their accuracies from the parameters of the "best" model.
- (9) Graphical display of the deformation model.

A detailed description of the above steps with practical examples can be found in Secord [1984]. Some applications will be given in case studies presented at this workshop. The reader is also referred to Chen [1983], Chrzanowski et al. [1983; 1985], and Chrzanowski and Secord [1983; 1985].

9. Computer Program Cluster "UNB DEFNAN"

The UNB generalized method has been implemented through software developed in FORTRAN 77 on an IBM 3090 mainframe and on an IBM PC/AT. The cluster of programs has the same behaviour in either system with the only variations being in file management and in graphical display. Hence, the following description of the modules is applicable to either system.

The cluster is most easily described with reference to Figure 9.1 which shows the arrangement of modules.

Any campaign of measurement, I_j , however populated, is utilized. If the configuration of measured relationships is complete, then an adjustment is performed resulting in least-squares estimates, \hat{x}_j , under explicit minimal constraints in 1, 2, or 3 dimensions. The intention behind "UNB DEFNAN" has been to remain flexible enough to accept the estimated coordinates and their variance-covariance matrix from any style of adjustment program which is then received by module CORDIF.

Following from at least two campaign adjustments are the analysis of trend and the modelling using coordinate differences, dx_s , in program module CORDIF within which there are several submodules. The first, CORDIFD, initiates the comparison of a pair of campaigns by creating a set of displacements versus minimal explicit constraints. It is this upon which the datum independent weighted transformation and modelling are based. Also, CORDIFD allows the segregation of stations common to the two campaigns or of only those stations of interest, e.g., only the reference network stations. The displacements can be readily depicted against their respective ellipses at any desired α level through graphics packages on either system. The weighted transformation is performed by submodule CORDIFW producing a datum independent indication of trend which may be visualized as displacement vectors with ellipses at any specified α level. Any collection or arrangement of stations can be considered in the modelling which is done in a very flexible submodule CORDIFM. Any model can be accommodated, provided that the functional relationship has been coded. Full statistical testing of the model and its constituents and any desired characteristics may be derived and their significance levels determined. One example is linear homogeneous strain for which the basic parameters are ϵ_x , ϵ_y , ϵ_{xy} , ω with a possible a_0 and b_0 . From this, the maximal and minimal strains, ϵ_{max} and ϵ_{min} and their orientation plus the vector of relative rigid body movement, d with azimuth A_d , would be derived accompanied by their standard deviations and $(1-\alpha)$ levels.

Circumvention of a campaign adjustment, especially when not allowed by the lack of a substantial configuration, requires considering the observations themselves. This may be

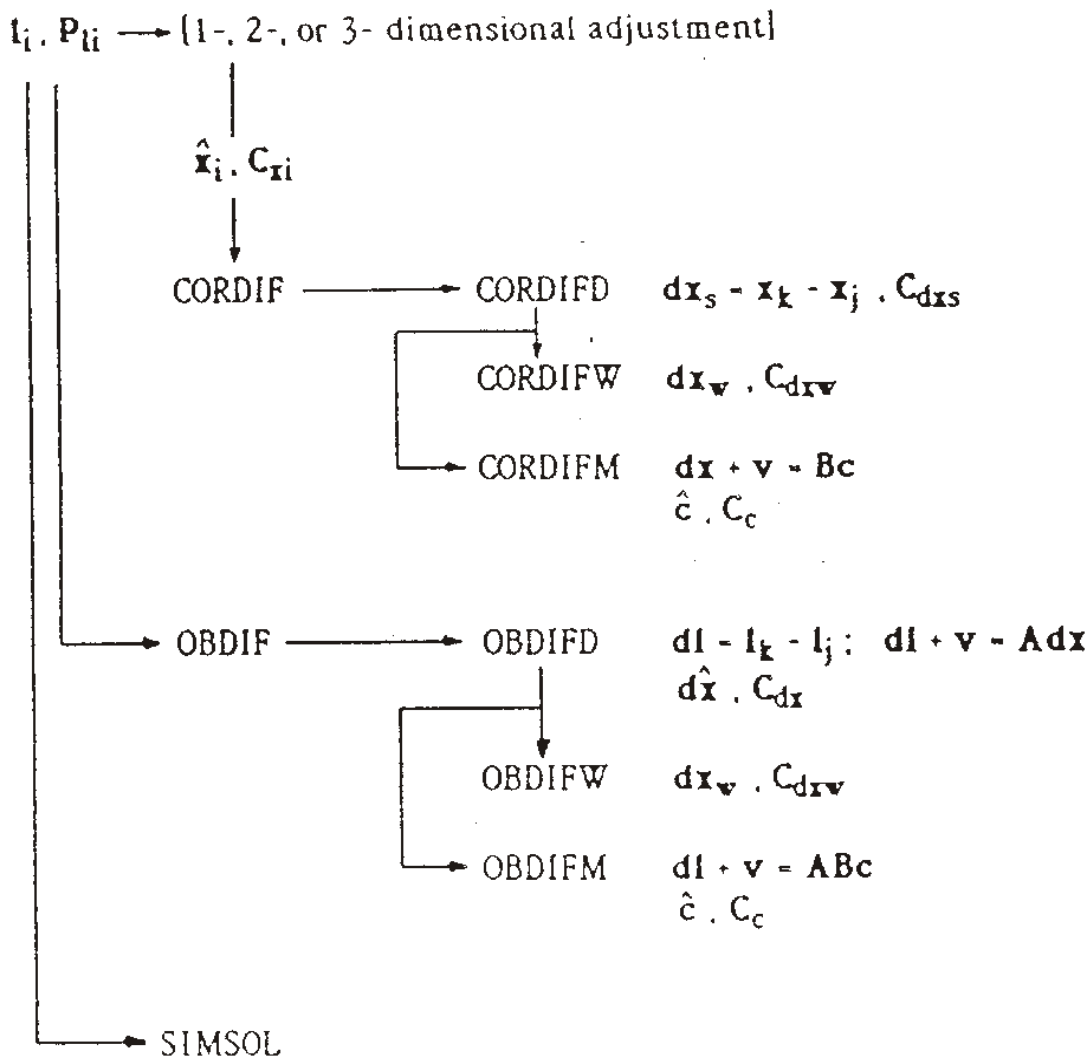


Figure 9.1 Program cluster "UNB DEFNAN."

readily done through module OBDIF which treats observation differences, $d\mathbf{l}$, in much the same way as the $d\mathbf{x}$ were treated in CORDIF with the additional flexibility of being capable of dealing with observables other than the customary geodetic angular and linear measurements. Displacements, $d\mathbf{x}$, can be estimated from the $d\mathbf{l}$ in submodule OBDIFD with results similar to those of CORDIFD, even with configuration and multiple datum defects associated with totally isolated but repeated observations. The weighted transformation, $d\mathbf{x}_w$, can be obtained through submodule OBDIFW for an indication of trend. Similarly as in CORDIFM, modelling may be done using the $d\mathbf{l}$ in submodule OBDIFM.

If the trend as indicated through the campaign comparisons would indicate the feasibility of a model considering all or many campaigns simultaneously, then this may be accomplished through module SIMSOL. This simultaneous solution can accommodate as many campaigns as desired with as few as one observation in a campaign. Rates, acceleration, and higher-order parameters may be estimated with full statistical assessment and the analysis of the observations.

Altogether, the program cluster "UNB DEFNAN" provides a flexible and versatile deformation analysis package which can also be utilized in the preanalysis and design of deformation monitoring schemes.

10. An Example of a Reference Geodetic Network

A pure triangulation network of 6 concrete pillar reference stations and 10 uniquely intersected dam crest points (Figure 10.1) was observed twice with 47 directions first and 53 directions in the second campaign. Least-squares estimations of the coordinates, \hat{x}_1 , \hat{x}_2 , were made under explicit minimal constraints involving stations 5 and 6 (considered as "fixed" and errorless) using UNB program GEOPAN (GEOdetic Plane adjustment and ANalysis). No observation in either campaign was detected as being an outlier under the τ max criterion at 0.95. The pooled variance factor, $\sigma_o^2 = 0.95278$, had $df = 31$ degrees of freedom. With each campaign having the same stations and datum, the observed displacement components d were obtained through the simple differencing of coordinates, through module CORDIFD, as

$$d = \hat{x}_2 - \hat{x}_1, \quad (10.1)$$

with cofactor matrix

$$Q_d = Q_1 + Q_2. \quad (10.2)$$

Within the d and Q_d are zero elements corresponding to the coordinates of the constraining stations 5 and 6.

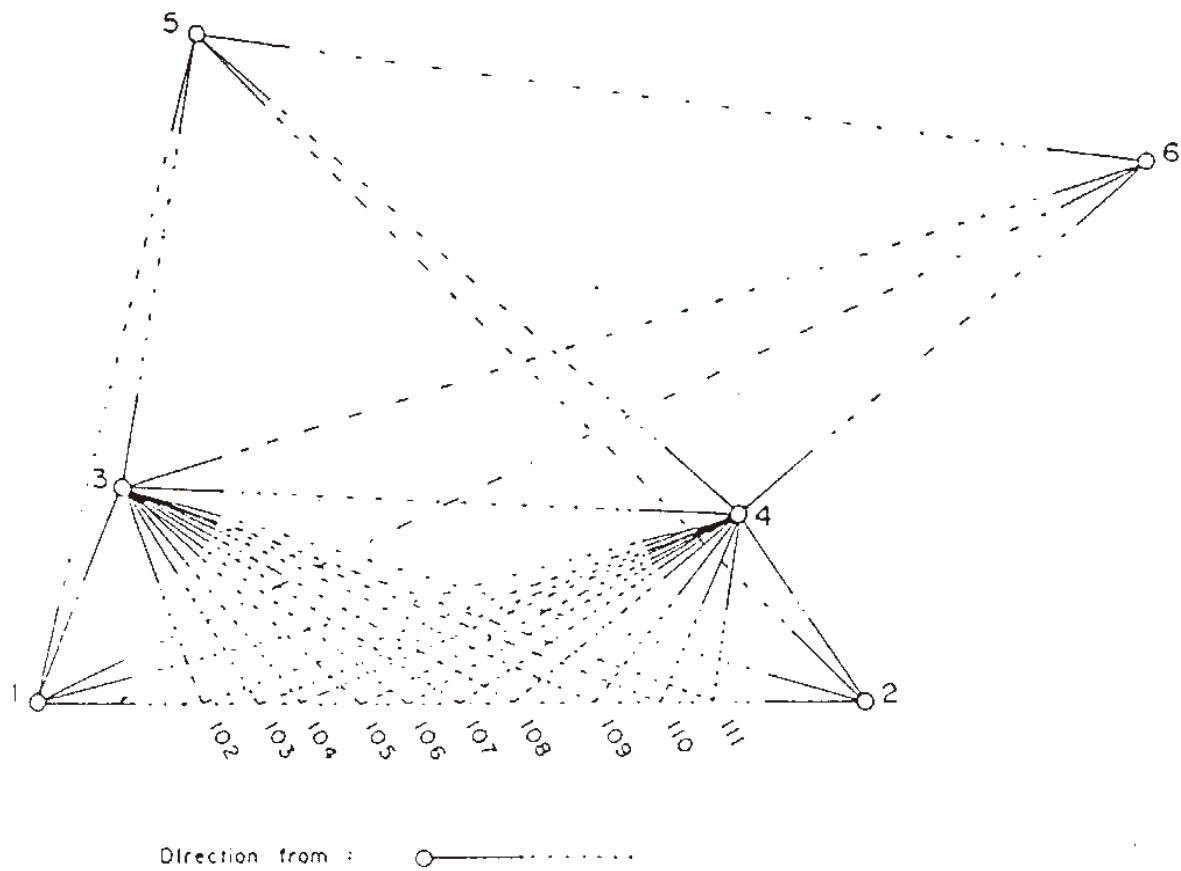


Figure 10.1 Reference geodetic network.

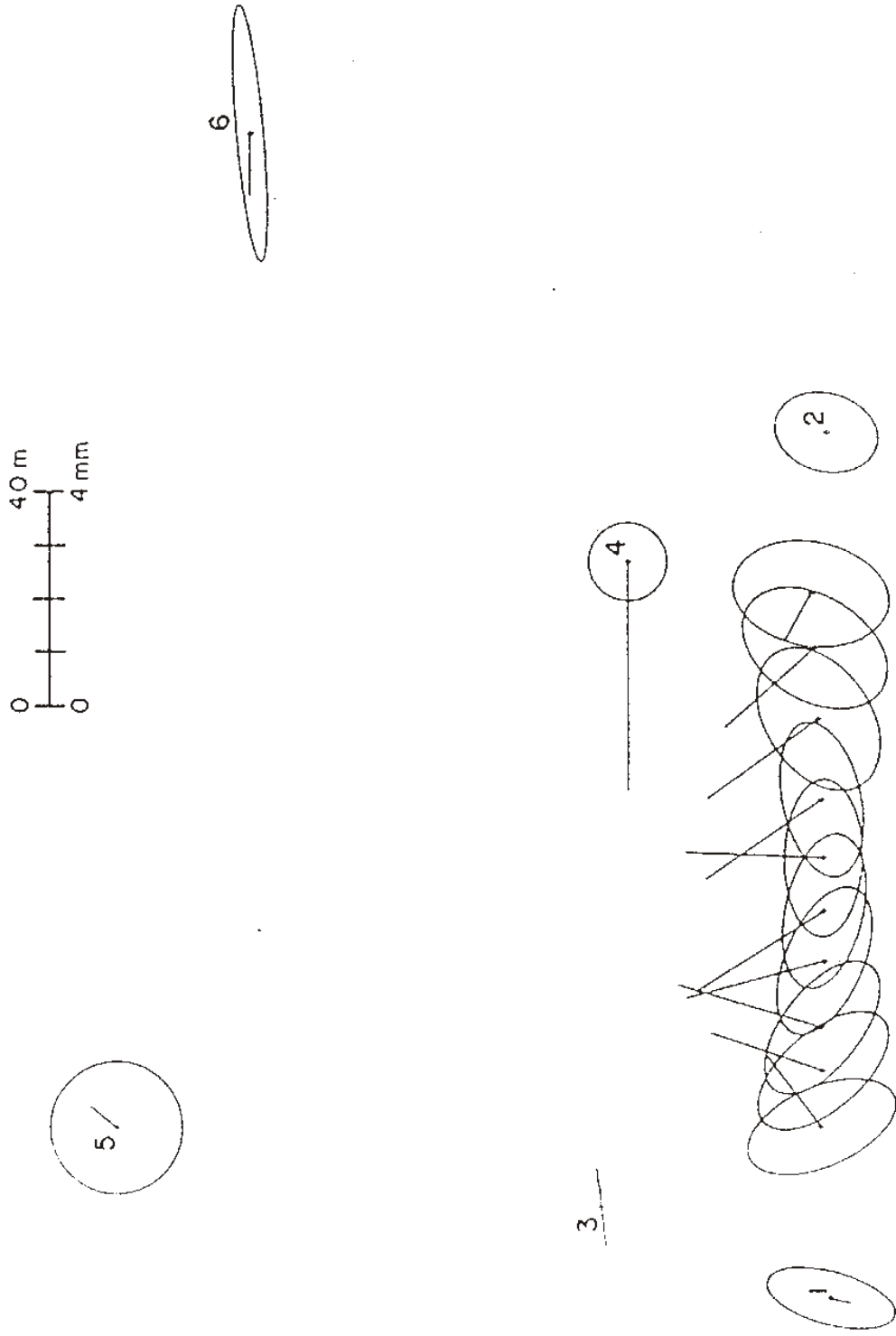


Figure 10.2 Displacements and confidence regions at $\alpha = 0.05$ after weighted transformation of station displacements.



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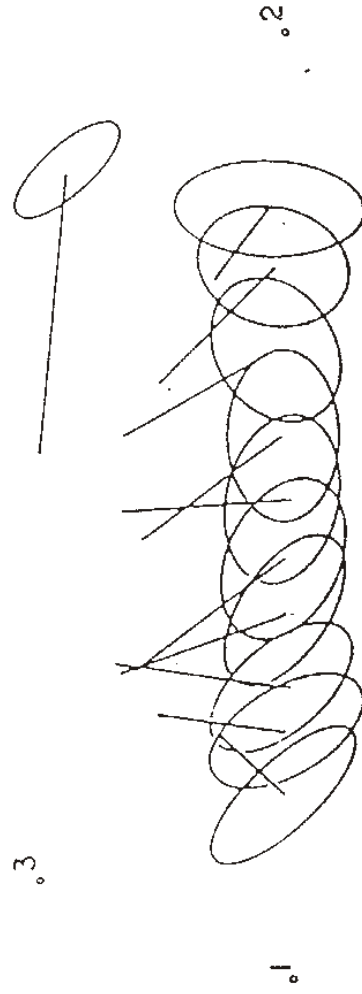


Figure 10.3 Displacements and confidence regions at $\alpha = 0.5$ derived from model of single point movement.

After the converged iteration of the weighted transformation of d from eqn. (10.1), the unique, datum independent displacement pattern produced by module CORDIFW using eqn. (5.2) is shown in Figure 10.2. Obviously, reference station 4 has moved significantly while the other reference stations remain stable at 0.95.

Having an a_i and b_i for each object point and also for station 4, and modelling the block of points 1, 2, 3, 5, and 6 as stable, the deformation model consisted of 22 parameters (i.e., 11 pairs of displacement components) which were estimated using module CORDIFM and eqn. (6.6). A plot of these displacements and their associated confidence regions at 0.95 is given in Figure 10.3. With 6 degrees of freedom in the modelling, the global test on the adequacy of the model was not rejected at 0.95 since:

$$T^2 = (\hat{\sigma}_{oc}^2) / (\sigma_o^2) = 1.7628 < F(6, 31; 0.05) = 2.41 .$$

Thus, with the a priori knowledge of the intention of the network serving as a reference, this model was adopted as appropriate.

11. Acknowledgements

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12. References

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APPENDIX I DETECTION OF OUTLIERS

There are two concepts of outlying observations in statistics. One is the "mean shift model," where an outlier has the distribution of $N(\mu + \lambda, \sigma^2)$ instead of $N(\mu, \sigma^2)$, and the other is the "variance-inflation model" in which an outlying observation is distributed as $N(\mu, a^2\sigma^2)$, $a^2 > 1$, i.e., its variance is larger than expected. Different strategies for reweighting observations in a least-squares adjustment for detection of outliers are based on the latter concept. In these notes, only the "mean shift model" will be discussed.

Consider the Gauss-Markoff model (l, Ax, σ^2Q) . Let an n vector of observations l be partitioned into two groups: l_1 and l_2 with l_1 being of dimension n_1 and free of outliers, and l_2 being of dimension n_2 and containing suspected outliers, denoted by δ . The mean shift model reads:

$$\begin{bmatrix} l_1 \\ l_2 \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} A_1 & 0 \\ A_2 & I \end{bmatrix} \begin{bmatrix} x \\ \delta \end{bmatrix}, \quad (I.1)$$

where v_i, A_i ($i=1,2$) are corresponding vectors of residuals and configuration matrices, respectively. More compactly, eqn. (I.1) is written as:

$$l + v = Ax + E\delta. \quad (I.1')$$

If the observations are uncorrelated, i.e., matrix Q is diagonal, model (1) is equivalent to the observation equations after outlying observations l_2 are removed. The statistical tests on outliers are to confirm, at a certain confidence level $(1-\alpha)$, the null hypothesis $H_0 : \delta = 0$ versus an alternative one $H_a : \delta \neq 0$. In the practice of outlier detection, an adjustment is performed with the original Gauss-Markoff model (l, Ax, σ_0^2Q) . The least-squares estimation of the residuals and their cofactor matrix is

$$\hat{v} = [A(A^TQ^{-1}A)^{-1}A^TQ^{-1} - I]l = Ml, \quad (I.2)$$

and

$$Q_{\hat{v}} = Q - A(A^TQ^{-1}A)^{-1}A^T, \quad (I.3)$$

respectively. The quadratic form of the residuals

$$R_1 = \hat{v}^TQ^{-1}\hat{v} \quad (I.4)$$

follows a $\sigma_0^2\chi^2$ distribution with degrees of freedom $df = n - \text{rank}(A)$. Introducing vector δ in model (I.1') will result in a reduction in the quadratic form of the residuals

$$\Delta R = R_1 - R_0 - \hat{v}^T Q^{-1} A_2 (A_2^T Q^{-1} Q \hat{v} Q^{-1} A_2)^{-1} A_2^T Q^{-1} \hat{v}, \quad (I.5)$$

which will be non-centrally $\sigma_0^2 \chi^2$ distributed with degrees of freedom n_2 if the null hypothesis is to be rejected. The quantity ΔR is statistically independent of $(R_1 - \Delta R)$. If there are no other suspected outliers in the observations, then $(R_1 - \Delta R)$ will follow a central $\sigma_0^2 \chi^2$ distribution with degrees of freedom being $(df - n_2)$. Confirmation of the suspected outliers is made:

$$(i) T_1 = \Delta R / n_2 \sigma_0^2 \geq F(\alpha; n_2, \infty), \quad (I.6)$$

if the a priori variance factor σ_0^2 is available;

$$(ii) T_2 = \Delta R / n_2 \tilde{\sigma}_0^2 \geq F(\alpha; n_2, df - n_2), \quad (I.7)$$

if the a posteriori variance factor σ_0^2 is used and estimated from

$$\tilde{\sigma}_0^2 = (R - \Delta R) / (df - n_2); \quad (I.8)$$

$$(iii) T_3 = \Delta R / n_2 \hat{\sigma}_0^2 \geq (df F(\alpha; n_2, df - n_2)) / ((df - n_2) + F(\alpha; n_2, df - n_2)), \quad (I.9)$$

if the a posteriori variance factor is computed from $\hat{\sigma}_0^2 = R / df$.

It is important to point out that the conclusions about outliers using the tests (I.7) and (I.9) are identical, because expression (I.9) can be derived directly from expression (I.7).

As a special case, if only one outlier is suspected, say, the i th observation, matrix A_2 in eqn. (I.5) is replaced by vector e_i , which is an n -vector with a unit value in the i th position and zeros elsewhere. Then eqn. (I.5) is reduced to

$$\Delta R_i = (e_i^T Q^{-1} \hat{v})^2 / e_i^T Q^{-1} Q \hat{v} Q^{-1} e_i. \quad (I.10)$$

In addition, if the observations are statistically independent, i.e., matrix Q is diagonal, then the expression ΔR_i is further simplified as

$$\Delta R_i = \hat{v}_i^2 / q_{vi}, \quad (I.11)$$

where q_{vi} is the i th diagonal element of Q_v , and v_i is the i th component of v . In the case of one suspected outlier, the statistical tests (I.6), (I.7), and (I.9) become the w -test [Kok, 1984; Baarda, 1968]:

$$w_i = \sqrt{\Delta R_i / \sigma_0^2} \geq \sqrt{F(\alpha; 1, \infty)} = n(\alpha/2), \quad (I.12)$$

the t -test [Heck, 1981]

$$t_i = \sqrt{\Delta R_i / (\tilde{\sigma}_0^2)_i} \geq \sqrt{F(\alpha; 1, df - 1)} = t(\alpha/2; df - 1), \quad (I.13)$$

and the τ -test [Pope, 1976]

$$\tau_i = \sqrt{\Delta R_i / \tilde{\sigma}_0^2} \geq \sqrt{(df F(\alpha; 1, df - 1)) / ((df - 1) + F(\alpha; 1, df - 1))} = \tau(\alpha/2; df), \quad (I.14)$$

respectively. $(\tilde{\sigma}_0^2)_i$ in eqn. (I.13) is calculated from

$$(\tilde{\sigma}_0^2)_i = (\hat{v}^T Q^{-1} \hat{v} - \Delta R_i) / (df - 1). \quad (I.15)$$

The τ -distribution is not very popular in statistics and no tabulated critical values are available, but they can easily be calculated by comparing the expressions (I.13) and (I.14)

as

$$\tau(\alpha/2; df) = \sqrt{df} \cdot t(\alpha/2; df-1) / \sqrt{(df-1) + t^2(\alpha/2; df-1)} . \quad (I.16)$$

Since the statistical tests (I.7) and I.9) are equivalent, so are the τ -test and the t-test.

The difficulty in the detection of outliers lies in the localization of outlying observations, especially when multiple outliers are present. An efficient strategy has been developed, and the interested readers are referred to Chen et al. [1986].

APPENDIX II

ASSESSMENT OF OBSERVATIONS

In the above discussions, the variance-covariance matrix of the observations or the weight relationship among the observations is assumed to be known. This, however, may not be the case in many practices, especially in heterogeneous networks. Assessment of the observations may have to be performed. The technique of MINQE (minimum norm quadratic estimation) provides a tool to estimate variance-covariance components.

Consider the linear Gauss-Markoff model (l, Ax, C) . Assume that the variance-covariance matrix, C , can be decomposed as

$$C = \sum_1^k \theta_i T_i, \quad (II.1)$$

where T_i are known matrices, θ_i are variance-covariance components to be estimated. Applying the MINQE principle, the $\theta = (\theta_1, \theta_2, \dots, \theta_k)^T$ can be estimated from

$$\theta = S^{-1} q, \quad (II.2)$$

where the (i,j) th element of matrix S is

$$s_{ij} = \text{tr}\{R T_i R T_j\}, \quad (II.3a)$$

and the i th component of vector q is

$$q_i = l^T R T_i R l, \quad (II.3b)$$

and

$$R = C^{-1}[I - A(A^T C^{-1} A)^{-1} A^T C^{-1}]. \quad (II.3c)$$

Since C is unknown, an iterative computation procedure has to be conducted. Let $\theta_i^{(0)}$, $\forall i$ be the a priori value of θ_i , then C in the above formulae is replaced by

$$C(\theta_i) = \sum_1^k \theta_i^{(0)} T_i.$$

From eqn. (II.2), θ is estimated and used as a priori values in the second iteration. If the process continues and the solution for θ converges, the final estimation of θ is independent of the selection of a priori values $\theta_i^{(0)}$, $\forall i$. This technique is called the iterated MINQE. For a detailed theoretical discussion and application, readers are referred to Chen [1983] and Chen and Chrzanowski [1985].